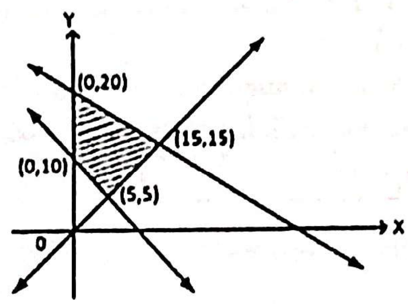
CHAPTER-12: LINEAR PROGRAMMING PROBLEMS

- **Linear Programming** is the process used to obtain minimum or maximum value of the linear function under known linear constraints.
- **Objective Function:** Linear function $z = ax + by$ where a and b are constants, which has to be maximized or minimized is called a linear objective function.
- **Constraints:** The linear inequalities or in equations or restrictions on the variables of a linear programming problem. The condition $x \geq 0$, $y \geq 0$ are known as non-negative restrictions.
- **Feasible Region:** It is defined as a set of points which satisfy all the constraints.
- To find feasible Region: Draw the graph of all the linear in equations and shade common region determined by all the constraints.
- **Feasible solutions:** Points within and on the boundary of the feasible region represents feasible solutions of the constraints.
- **Optimal feasible solution:** Optimal feasible solution which optimizes the objective function is called optimal feasible solution.

MULTIPLE CHOICE QUESTIONS

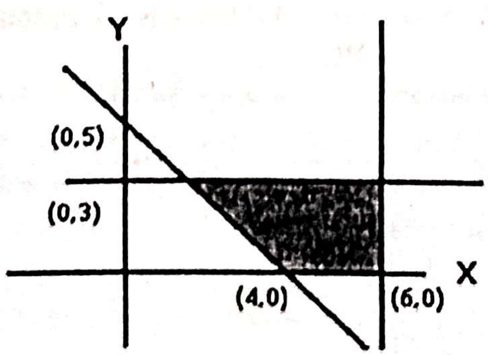
1. The feasible region of an LPP is shown in the figure. If $Z=3x+9y$, then the minimum value of z occurs at



(a) (0,20) (b) (0,10) (c) (5,5) (d) (15,15) Sol: Option: (c)

Corner Points	Z=3x+9y
(0,20)	180
(15,15)	180
(5,5)	60-MIN
(0,10)	90

2. The constraints of the LPP represented by the following figure is



(a) $5x + 4y \geq 20, x \leq 6, y \geq 3, x \geq 0, y \geq 0$
(b) $5x + 4y \leq 20, x \leq 6, y \geq 3, x \geq 0, y \geq 0$
(c) $5x + 4y \geq 20, x \leq 6, y \leq 3, x \geq 0, y \geq 0$
(d) $5x + 4y \geq 20, x \geq 6, y \leq 3, x \geq 0, y \geq 0$ Sol : Option: (c)

3. If an LPP admits optimal solution at two consecutive vertices of a feasible region, then
(a) the LPP under consideration is not solvable
(b) the LPP under consideration must be reconstructed
(c) the required optimal solution is at the mid-point of the line joining two points
(d) the optimal solution occurs at every point on the line joining these two points
Option : (d)

4. Which of the following statement is correct
(a) every LPP admits an optimal solution
(b) If a L.P.P admits two optimal solutions it has an infinite number of optimal solutions
(c) A L.P.P admits unique optimal solution
(d) (0,0) is the only optimal solution. Option : (b)

5. Minimum value of $z = x-5y+20$ subject to the constraints:
$x - y \geq 0, -x + 2y \geq 2, x \geq 3, y \leq 4$ is given by
(a) (-6) at (4,6) (b) 4 at (4,4) (c) (-9) at (4,3) (d) 6 at (4,5)

Corner Points	Z=x-5y+20
(4,6)	-6=minimum
(4,4)	4
(4,3)	9
(4,5)	-1

So, Option: (a)

6. The corner points of the bounded feasible region determined by a system of linear constraints are (0,3), (1,1) and (3,0). Let $Z = px + qy$, where $p, q > 0$. The condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is:
(a) $p = 2q$ (b) $p = q/2$ (c) $p = 3q$ (d) $p = q$
Sol:3p=p+q implies 2p=q So Option : (b)

7. Which of the points A(80,10), B(20,20), C(60,60) D(20,25) lie in the feasible region of the constraints given below. $x + 5y \leq 200$, $2x + 3y \leq 134$, $x \geq 0$, $y \geq 0$
(a) Points A,B,and C only (b) Points B,C and D only (c) only points A and D (d) only points B and D
Sol: Only B and D satisfies all constraints, so Option : (d)

8. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4,6) and minimum value 19 at (3,2). Which of the following is true?
(a) $a = 9, b = 1$ (b) $a = 5, b = 2$ (c) $a = 3, b = 5$ (d) $a = 5, b = 3$
Sol: 4a+6b=42 & 3a+2b=19 on solving a=5, b=2 So, Option (b)

ASSERTION-REASONING QUESTIONS

Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A)
(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

1. ASSERTION: Maximum value of $z=11x+5y$ subject to

$3x+2y\le 25, x+y\le 10$ & $x,y\ge 0$ with corner points

(0,10), (5,5) and (25/3,0), occurs at $\left(\frac{25}{3},0\right)$

REASON: If the feasible region of the LPP is bounded then the maximum & minimum value of the objective function occurs at corner points.

Ans: (a) as both A and R are true and R is the correct explanation for A.

2. ASSERTION: If the open half plane represented by $ax + by > M$ has no point common with the unbounded feasible region then M is the maximum value of z otherwise z has no maximum value.

REASON: A solution that also satisfies the non-negativity restriction of a LPP is called the feasible region.

Ans: (b) Both A and R are true but R is not correct explanation for A

3. ASSERTION: Feasible region of all LPP are convex polygon.

REASON: A polygon is said to be a convex polygon, if the line segment joining any two points of the polygon is completely contained in the polygon.

Ans: (d) as A is false and R is true.

4. ASSERTION: Value of the objective function if it exists is always positive.

REASON: Feasible region of any LPP lies only in the first quadrant as it involves non negative restrictions.

Ans(d) as value of objective function is any real number, so A is false but R is true.

5. ASSERTION: For the bounded feasible region of a LPP if minimum value of the objective function exists at two consecutive corner points, then the LPP has infinitely many optimaol solution.

REASON: If minimum value exists at two consecutive vertices of a bounded feasible region, then every point of the segment joining those two vertices will give same value for the objective function.

Ans: (a) Both A and R are true and R is the correct explanation for A.

6. ASSERTION: The half plane $2x+3y\le 6$, contains (1,1) and (3,0)

REASON: Any point if it satisfies the inequality $ax+by\le c$, should lie in the half plane described by it.

Ans: (a), as both A and R are true and R is correct explanation for A.

7. ASSERTION: Every feasible solution of a LPP is an optimal solution of the LPP

REASON: Feasible solution is a point of feasible region that satisfy all constraints including non negative constraints of LPP

Ans: (d) A is false but R is true.

8. ASSERTION: The point (2,-3) does not lie on the half plane $3x+2y\ge 10$

REASON: Every point of a half plane $3x+2y\ge 10$ should satisfy the inequation.

Ans: (a) both A and R are correct and R is correct explanation for A.

9. ASSERTION: Corner points of a feasible region are (0,6), (3,2) and (3,0) for a LPP with objective function $Z = 11x+7y$, and maximum value of Z is 47.

REASON: Optimal value of an objective function occurs only at corner points if they exist.

Ans: (a) Both A and R are correct and R is proper reason for the A.

10.ASSERTION: For a LPP, the feasible region is unbounded and minimum value of the objective function Z=px+qy occurs at (a,b) with value "m". Then optimal solution is x=a,y=b with optimal value "m".

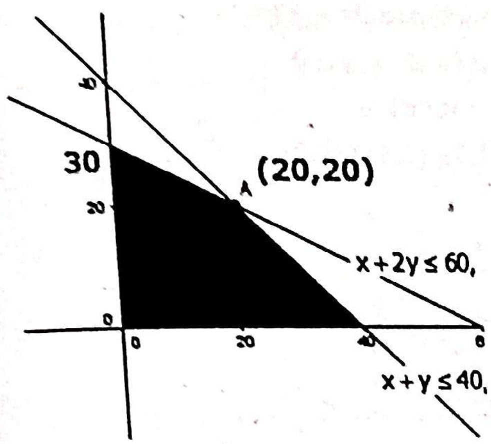
REASON:If feasible region is unbounded, it is to decide minimum exists or not after considering px+qy<m and whether this half plane has common points with feasible region of the problem.

Ans: (c) as A is wrong R is correct.

VERY SHORT ANSWER QUESTIONS

1. Find the maximum value of z=3x+4y subject to the constraints

$x+y \le 40,\ x+2y \le 60,\ x,y \ge 0$

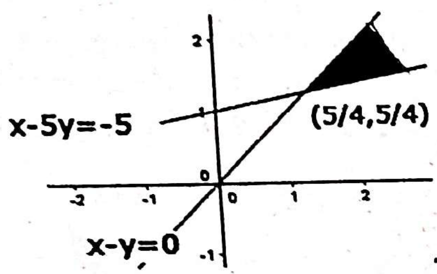


CORNER POINTS	Z=3X+4Y
(0,30)	120
(20,20)	140
(40,0)	120
(0,0)	0

Max z is 140 at (20,200

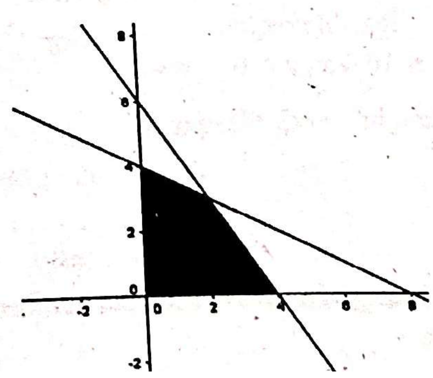
2. If the minimum value of the objective function z=2x+10y exists subject to constraints

$x-y \ge 0,\ x-5y \le -5,\ x,y \ge 0$ find it.



Minimum value is 15 at (5/4,5/4).

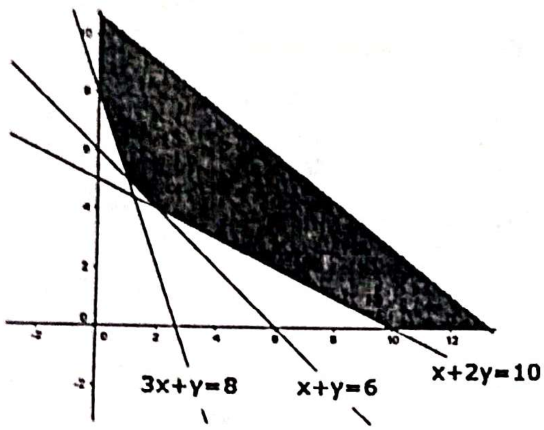
3. Shown below is the feasible region of maximization problem whose objective function is given by z=5x+3y.



(i) List all the constraints the problem is subjected to.
(ii) Find the optimal solution of the problem.
Constraints are $x + 2y \leq 8$, $3x + 2y \leq 12$ & $x, y \geq 0$

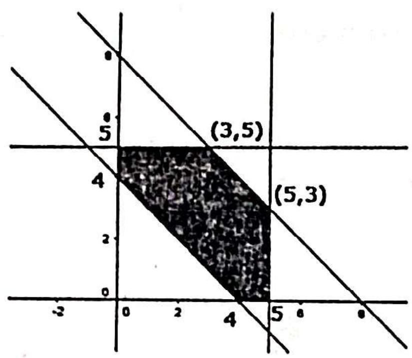
Corner Points	Z=5x+3y
(0,0)	0
(2,3)	19
(0,4)	12
(4,0)	20=Maximum

4. For the feasible region of a LPP given below, write all constraints and corner points of the feasible region.



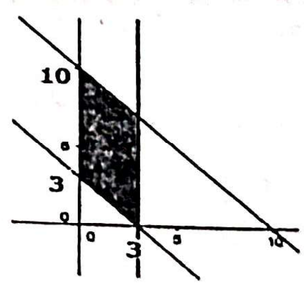
Constraints are $3x + y \geq 8$, $x + y \geq 6$, $x + 2y \geq 10$ & $x, y \geq 0$
Corner Points are
(0,8), (1,5), (2,4) (10,0)

5. If the objective function of the LPP is z=x-7y+200, feasible region is given below.



Find the difference between the optimal values of Z.
Sol: Max=205 , Min=165, Difference=205-165=40

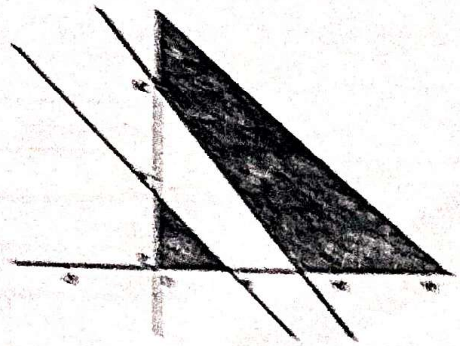
6. What are the constraints of the LPP, whose feasible region is given below, also find the area of the feasible region



Constraints are $x + y \geq 3, x + y \leq 10$ & $x, y \geq 0$

Area of Parallelogram=base X height =7x3=21sq.units

7. The feasible region of a LPP is as given below. Find the number of optimal solutions for the LPP and justify your answer.



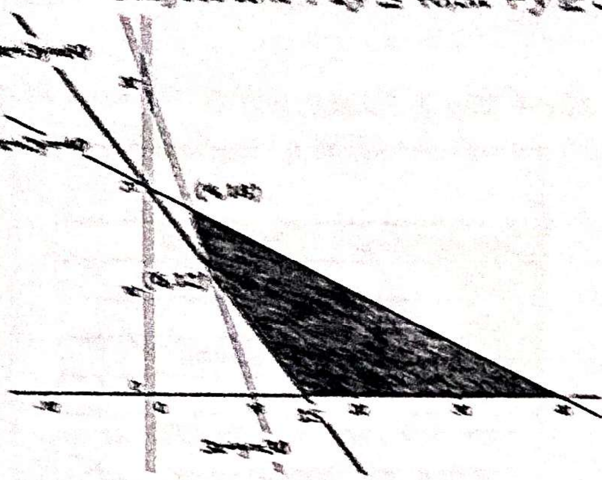
Sol: Zero, as there is no region satisfy all constraints of the problem.

SHORT ANSWER QUESTIONS-SOLVED

1. Minimize $Z = 20x + 10y$

Subject to $x + 2y \leq 40, 3x + y \geq 30, 4x + 3y \geq 60, x, y \geq 0$

Sol:



Corner Pts	Z=20x+10y
(15,0)	300
(40,0)	800
(4,18)	260
(6,12)	240

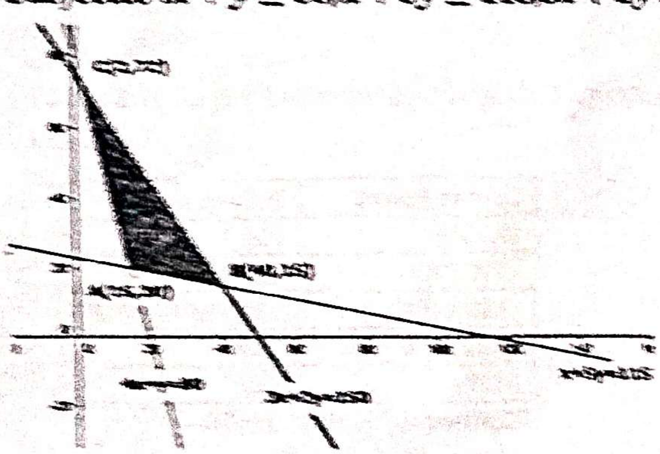
Minimum Value of Z is 240 at (6,12)

Hence x=6,y=12 is the optimal solution

and the optimal value of Z is 240.

2. Minimize $Z = 6x + 3y$

Subject to $4x + y \geq 80, x + 5y \geq 115, 3x + 2y \leq 150, x, y \geq 0$

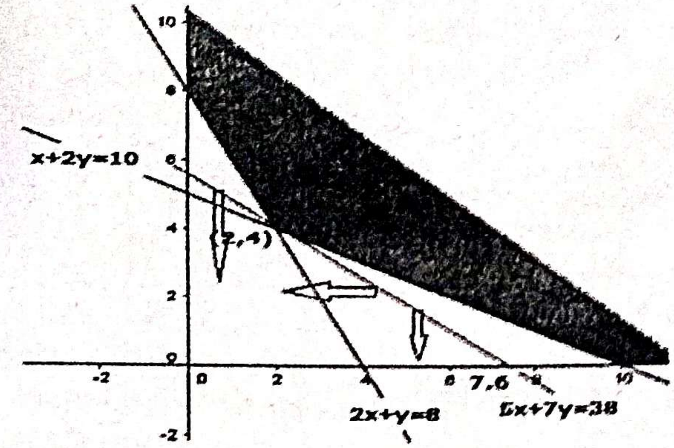


Corner pts	Z=6x+3y
A(15,20)	150
B(40,15)	285
C(2,72)	228

Minimum value of Z is 150 at x=15,y=20

3. Minimize $Z = 5x + 7y$

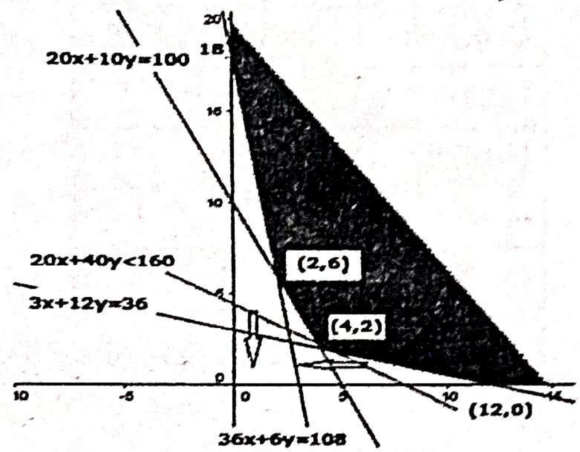
Subject to $2x + y \geq 8, x + 2y \geq 10, x, y \geq 0$



CORNER PTS	Z=5x+7y
(0,8)	56
(2,4)	38 -MINIMUM
(10,0)	50

Feasible Region and halfplane of 5x+7y<38 has no common points, so Minimum value of Z is 38 at (2,4).

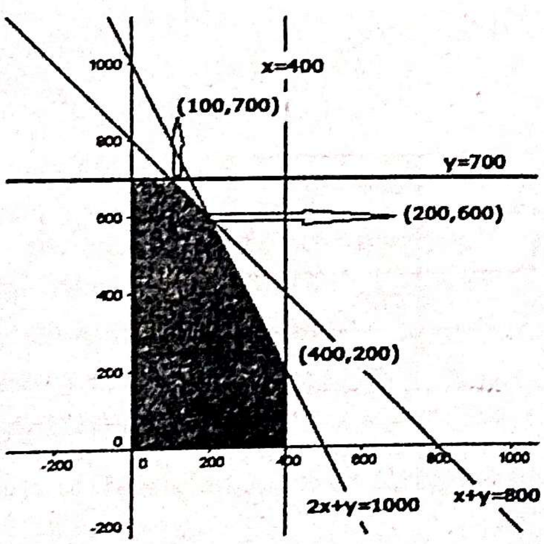
6. $Minimize Z = 20x + 40y$
$Subject to 36x + 6y \geq 108, 3x + 12y \geq 36, 20x + 10y \geq 100 \& x, y \geq 0$



Corner Points	Z=20x+40y
(0,18)	720
(2,6)	280
(4,2)	160=Min
(12,0)	240

Here, Half plane 20x+40y<160 has no common with feasible region, so, Min Z is 160 at x=4, y=2.

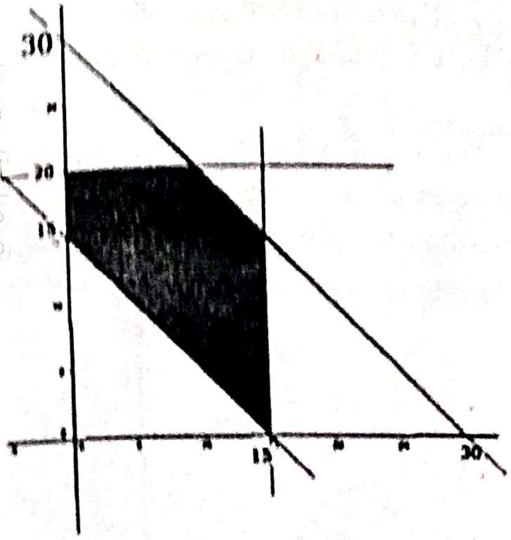
7. $Maximize Z = 2x + 1.5y$
$Subject to 2x + y \leq 1000, x + y \leq 800, 0 \leq x \leq 400, 0 \leq y \leq 700$



Corner Points	Z=2x+1.5y
(0,700)	1050
(100,700)	1250
(200,600)	1300=MAXIMUM
(400,200)	1100
(400,0)	800

Optimal Sol: x=200, y=600
Optimal Value:1300

8. For a LPP, the feasible region is as given below. Objective function is Minimize Z=30x-30y+1800. Write all constraints of the LPP and find the optimal value of the objective function and optimal solution also.



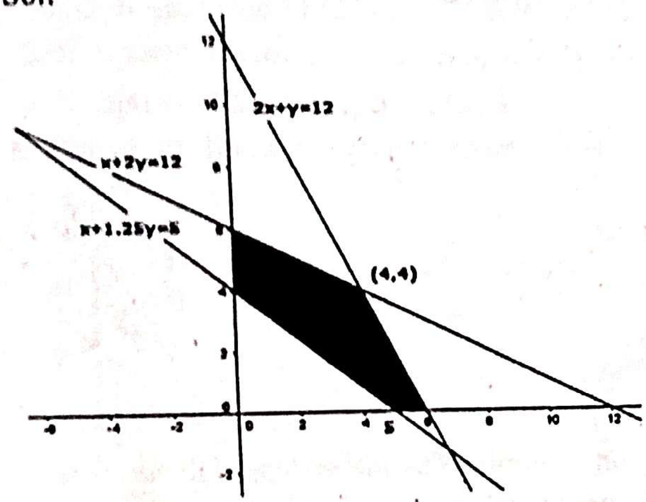
Corner Points	Z=30x-30y+1800
(0,15)	1350
(0,20)	1200=Minimum at x=0, y=20
(15,0)	2250
(10,20)	1500
(15,15)	1800

Sol : Constraints are $x+y \geq 15$, $x+y \leq 30$, $0 \leq x \leq 15, 0 \leq y \leq 20$

9. Maximize $Z = 600x + 400y$

Subject to $x + 2y \leq 12, 2x + y \leq 12, x + 1.25y \geq 5$ & $0 \leq x, 0 \leq y$

Sol:

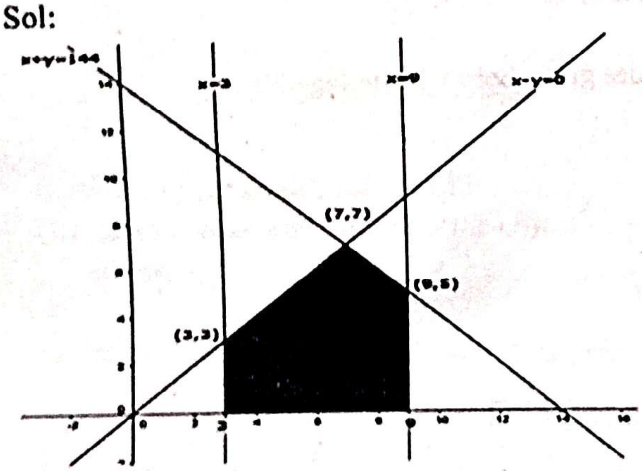


Corner Points	Z=600x+400y
(5,0)	3000
(6,0)	3600
(4,4)	4000=Maximum
(0,6)	2400
(0,4)	1600

Optimal Sol: x=4, y=4

10. Minimize & Maximize $Z = 2x + y$; Subject to $x - y \geq 0, x + y \leq 14, 3 \leq x \leq 9, y \geq 0$

Sol:



Corner Points	Z=2x+y
(3,0)	6=Minimum
(9,0)	18
(9,5)	23=Maximum
(7,7)	21
(3,3)	9

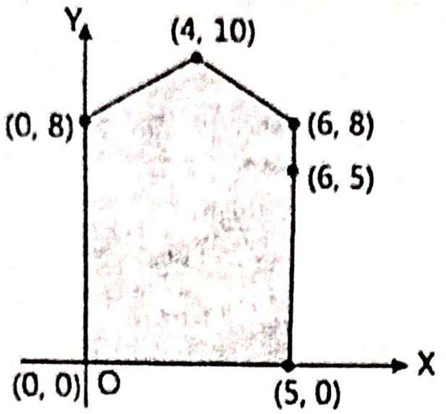
Optimal solutions;

For Minimum ; x=3, y=0

for maximum ; x=9, y=5

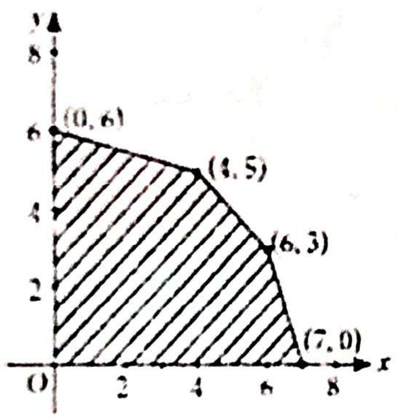
CASE STUDY BASED QUESTIONS

1. The feasible region for a LPP is as shown below, $Z=3x-4y$
   (a) Find the points at which maximum and minimum of Z occurs.
   (b) Find the difference between the optimum values of Z.



Ans: (a) Minimum Value is -32 at (0,8) Maximum Value is 15 at (5,0)
(b) Difference is 47

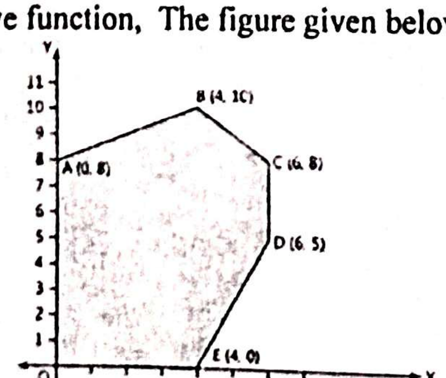
2. On the basis of the feasible region of a LPP as given below,
(a) Find the optimal value and solution of the objective function $Z=2x+5y$



(b) In general, if the corner points of the feasible region determined by the system of linear constraints are (0,10), (5,5),(15,15),(0,20) and $Z= px+qy$, where $p, q >0$. Find the relation between p and q so that the maximum occurs at (15,15) and at (0,20)

Sol: (a) Minimum value is 0 at (0,0) Maximum Value is 33 at (4,5)
(b) $15p+15q=20q$ implies $q=3p$

3. Let $Z=4x-7y$ be the objective function, The figure given below is the feasible region.



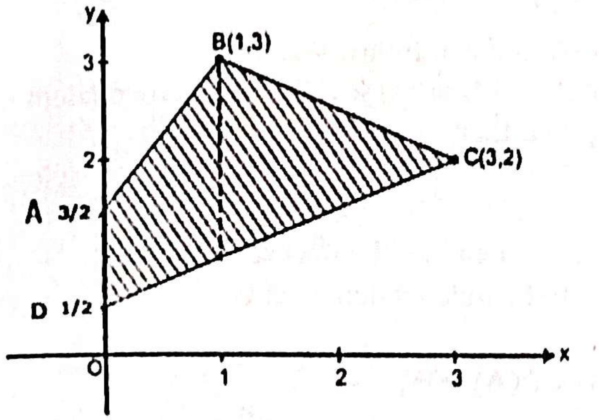
(a) Find the maximum and minimum value of Z and also the optimal solutions.

(b) Let Z=px+qy, where p,q>0 be the objective function. Find the condition on p and q so that the maximum value at B and C are equal. Also mention the number of optimal solutions in this case.

Sol: (a) Minimum Value is (-56) at (0,8) Maximum Value is 16 at (4,0)

(b) 4p+10q=6p+8q implies p=q, in this case, there are infinitely many optimal solution; all points of the segment BC are optimal solutions.

4. Given below is the feasible region of LPP with constraints



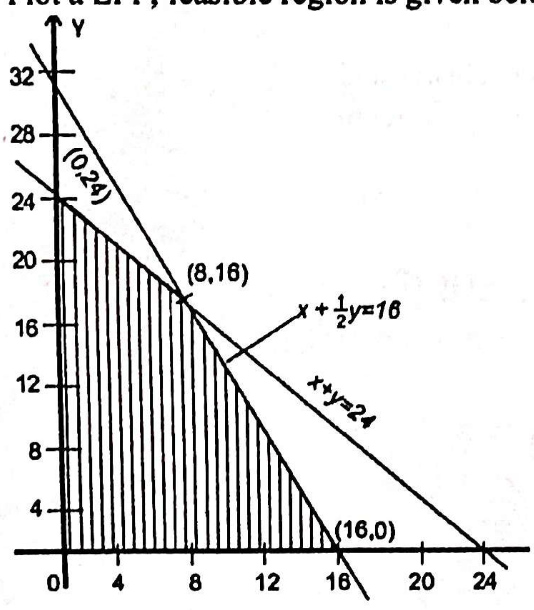
(a) Write all constraints of LPP

(b) Optimal value Maximize Z=5x-2y and corresponding Optimal point.

Sol: Constraints are $2y-3x\leq 3,\ x+2y\leq 7,\ 2y-x\geq 1\ \&\ x,y\geq 0$

Optimal Value is 11 at C(3,2)

5. Flot a LPP, feasible region is given below.



(a) write all constraints of the LPP

(b) find the maximum value of the objective function Z =3x+8y and also find the optimal solution.

Sol:

Constraints are $x+y\leq 24, 2x+y\leq 32\ \&\ x,y\geq 0$ Maximum Z=192 at (0,24)

CHAPTER-13-PROBABILITY

Definitions and Formulae:

Conditional Probability: If A and B are two events associated with any random experiment, then P(A/B) represents the probability of occurrence of event A knowing that the event B has already occurred.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0.$$

$P(B) \neq 0$,, means that the event B should not be impossible.

Multiplication Theorem on Probability: If the event A and B are associated with any random experiment and the occurrence of one depends on the other, then

$P(A \cap B) = P(A).P(B/A)$, where $P(A) \neq 0$

Independent Events:

When the probability of occurrence of one event does not depend on the occurrence /non-occurrence of the other event then those events are said to be independent events.

Then P(A/B)=P(A) and P(B/A)=P(B)

So, for any two independent events A and B, $P(A \cap B) = P(A).P(B)$.

Theorem on total probability:

$If E_i (i = 1,2,3,\ldots n) be a partition of sample space and all E_i have non-zero proabability. A be any event associated with the sample space, which occurs with E_1 or E_2 or E_3 or \ldots or E_n then$

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + \ldots + P(E_n)P(A/E_n)$$

Bayes' Theorem:

Let S be the sample space and $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or,..or E_n then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

MULTIPLE CHOICE QUESTIONS

1. For any two events A and B, P(A')=1/2, P(B')=2/3 and $P(A \cap B)$=1/4, then $P(A'/B')$ equals:

   (a) 8/9 (b) 5/8 (c) 1/8 (d) ¼

   Ans: (b)

   $$P(A) = 1/2, P(B) = 1/3$$
   $$P(AUB) = 1/2 + 1/3 - 1/4 = 7/12$$
   $$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(AUB)}{P(B')} = \frac{1 - 7/12}{2/3} = \frac{5/12}{2/3} = 5/8$$

2. For any two events A and B, $P(A) = 4/5$ and $P(A \cap B) = 7/10$, then $P(B/A)$ is

   (a) 1/10 (b) 1/8 (c) 17/20 (d) 7/8

   Ans: (d) $P(B/A) = \frac{7/10}{4/5} = 7/8.$

3. If A and B are two independent events such that P(A)=1/2 and P(B)=1/4, then P(B'/A) is

   (a) 1/4 (b) 3/4 (c) 1/8 (d) 1

   Ans: (b) $P(B'/A) = \frac{P(B' \cap A)}{P(A)} = \frac{P(B')P(A)}{P(A)} = P(B') = 3/4$

4. If two events A and B, P(A-B)=1/5 and P(A)= 3/5, then P(B/A) is equal to

   (a) ½ (b) 3/5 (c) 2/5 (d) 2/3

Ans: (d) $P(A - B) = 1/5, P(A) = 3/5$

$$P(A \cap B) = 3/5 - 1/5 = 2/5$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{2/5}{3/5} = 2/3.$$

5. If P(A/B)=0.3, P(A)= 0.4 and P(B)=0.8, then P(B/A) is equal to

(a) 0.6 (b) 0.3 (c) 0.06 (d) 0.4

Ans: (a) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0.8} = 0.3$

$$P(A \cap B) = 0.24$$

$$P(B/A) = 0.24/0.4 = 0.6$$

6. If A and B are two events such that P(A/B)=2.P(B/A) and P(A)+P(B)=2/3, then P(B) is

(a) 2/9 (b) 7/9 (c) 4/9 (d0 5/9.

Ans: (a) $\frac{P(A \cap B)}{P(B)} = 2\frac{P(A \cap B)}{P(A)}$

$$P(A) = 2P(B)$$

$$3P(B) = 2/3$$

$$P(B) = 2/9$$

7. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is

(a) 1/9 (b) 4/9 (c) 1/18 (d) 1/2

Ans: (d) $A = Sum9 = \{(3,6), (4,5), (5,4), (6,3)\}$

$B = onedieshows4 = \{(1,4), (2,4), (3,4), (5,4), (6,4), (4,1), (4,2), (4,3), (4,5), (4,6)\}$

$P(A) = 4/36 P(B) = 10/36$

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{2/36}{4/36} = 1/2$$

8. Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is

(a) 27/32 (b) 5/32 (c) 31/32 (d) 1/32

Ans: (c) P(atleast one H) = 1 − P(none is H)

$= 1 - 1/32 = 31/32.$

9. Ramesh can hit a target 2 out of 3 times. He tried to hit the target twice. The probability that the he missed the target exactly once is

(a) 2/3 (b) 1/3 (c) 4/9 (d) 1/9

Ans: (c) P(A) = 2/3 P(A') = 1/3 (A − hit, A' − not hit)

P(only once hit) $= \frac{2}{3}.\frac{1}{3} + \frac{1}{3}.\frac{2}{3} = 4/9.$

10. Three dice are thrown simultaneously. The probability of the events that at least one six comes up is

(b) 27/216 (b) 55/216 (c) 91/216 (d) 1/216

Ans: (c) P(atleast one 6) = 1 − P(none is 6)

$= 1 - 125/216 = 91/216.$

ASSERTION AND REASON QUESTIONS

Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A)

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

1 **Assertion (A):** If A and B are two independent events with P(A)=1/5 and P(B)=1/5, then P(A'/B) is 1/5.

Reason (R) **:** $P(A'/B) = \frac{P(A'\cap B)}{P(B)}$ **Ans: (d)**

$$P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(A').P(B)}{P(B)} = P(A')$$

$$= 1 - P(A) = 4/5$$

So A is false and R isT rue.

2. **Assertion (A) :** Let A and B be two events such that P (A) =1/5 and P (A or B) =1/2 then P(B)=3/8 for A and B are independent events.

Reason (R) : For independent events P (A or B) = P (A) + P(B) - P(A).P(B).

Ans: (a) as A is true and R is the correct explanation for A.

ForAssertion:

$P(AUB) = 1/2$

$P(A) + P(B) - P(A).P(B) = 1/2$

$P(B) = 3/8(\because P(A) = 1 - 1/5 = 4/5)$

Riscorrectexxplanation.

3. **Assertion (A):** Two coins are tossed simultaneously. The probability of getting two heads, if it is know that at least one head comes up, is 1/3.

Reason (R) : Let E and F be two events with a random experiment, then

$$P(F/E) = \frac{P(E\cap F)}{P(E)}.$$

Ans: (a) *ForAssertion* $F = \{HH\} E = \{HH, HT, TH\}, P(F/E) = \frac{1/4}{3/4} = 1/3$

4. **Assertion (A) :** If A and B are mutually exclusive events with P (A') =5/6 and P (B)=1/3. Then P (A/B')=1/4.

Reason (R) : If A and B are two events such that P (A)=0.2, P(B)=0.6 and P(A/B)=0.2 then the value of P(A/B') is 0.2.

Ans: (b) as A is true and R is not the correct explanation for A.

ForAssertion: $P(A) = 1/6$

$$P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{1/6 - 0}{1 - 1/3} = 1/4$$

Re a son: $P(A \cap B) = 0.12$

$$P(A/B') = \frac{P(A\cap B')}{P(B')} = \frac{P(A)-P(A\cap B)}{1-P(B)} = \frac{0.08}{0.4} = 0.2$$

5. Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A).P(B)$

Assertion(A): $P(A/B)$=$P(A)$and$P(B/A)$=$P(B)$

Reason(R): $P(A \cup B)$=$P(A)$+$P(B)$

Ans: (c) as A is correct but R is false.

6. Let A and B be two independent events.

Assertion (A): If $P(A)$=0.3 and $P(A \cup \bar{B})$=0.8 then $P(B)$ is 2/7

Reason(R) : $P(\bar{E})$=1-$P(E)$, for any event E.

Ans: (a) as $P(AUB') = P(A) + P(B') - P(A).P(B')$

$P(B') = 5/7; P(B) = 2/7$

Hence A is true and R is the correct explanation for A.

Assertion (A): Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely.
If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively, then P(E/F)=1/4 and P(F/E)=1/13.
Reason (R): E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.

Ans: (b) as A is correct but R is not the correct explanation of A.

8. Consider that following statements:
Assertion (A): Let A and B be two independent events. Then P(A and B)= P(A)+P(B)
Reason (R): Three events A,B and C are said to be independent if
$P(A\cap B\cap C)=P(A).P(B).P(C)$

Ans: (d) as $P(A \text{ and } B)=P(A)+P(B)-P(A).P(B)$, hence A is false and R is True.

9. **Assertion (A):** In rolling a die, event A= {1, 3, 5} and event B = {2, 4} are mutually exclusive events.
Reason (R): In a sample space two events are mutually exclusive if they do not occur at the same time.
Ans: (a) A is true as $P(A\cap B)=\phi$ and R is the correct explanation of A.

10. For any two independent events A and B. P(A)=p and P(B)=q
Assertion (A): The probability that exactly one of the events A and B occurs is p+q-2pq
Reason (R): $P(A\cup B)=P(A)+P(B)-P(A)P(B)$ **Ans:** (b)
A is correct but R is not the correct explanation of A.

VERY SHORT ANSWER QUESTIONS

1. A pair of dice is thrown. If the two numbers appearing on them are different, find the probability that the sum of the numbers is 6.
Ans: A: Number appearing are different n(A)=30 (except (1,1),(2,2),(3,3),(4,4),(5,5) and (6,6))
B: Sum of the numbers is 6. P(A)=30/36
A and B={(1,5),(2,4),(4,2),(5,1)}
P(A and B)=4/36 $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{4/36}{30/36}=4/30$

2. In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is girl?
Ans: A: Student of Class XII B: The student is a girl.
n(A & B)=10% of 430=43. $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{43}{430}=\frac{1}{10}$

3. Two balls are drawn from a bag containing 2white, 3 red and 4 black balls one by one without replacement. What is the probability that at least one ball is red?
Ans: P(at least one red ball)=1-P(none of the ball is red)
(that is 1st ball is non red and 2nd ball is non red.) $=1-\frac{6}{9}.\frac{5}{8}=\frac{42}{72}=\frac{7}{12}$

4. If P(A)=3/8, P(B)=1/2 and P(A and B)=1/4, find $P(A'/B')$
Ans:

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(AUB)'}{1-P(B)} = \frac{1-P(AUB)}{1-P(B)}$$

$$P(AUB) = P(A) + P(B) - P(A \cap B)$$

$$= 3/8 + 1/2 - 1/4 = \frac{5}{8}, P(A'/B') = \frac{1-P(AUB)}{1-P(B)} = \frac{1-\frac{5}{8}}{1-\frac{1}{2}} = \frac{3/8}{1/2} = 3/4$$

5. A committee of 4 students is selected at random from a group of 8 boys and 4 girls. Given that there is at least one girl in the committee, calculate the probability that there are exactly 2 girls in the committee.

Ans:

A: at least one girl in the committee. B: exactly 02 girls in the committee.

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A) = 1 - P(none\ is\ girl) = 1 - \frac{^8C_4}{^{12}C_4} = 1 - \frac{70}{495} = \frac{85}{99}$$

$$P(A \cap B) = P(2G\ and\ 2B) = \frac{^8C_2 . {^4C_2}}{^{12}C_4} = \frac{28.6}{495} = \frac{56}{165}$$

6. Events E and F are independent. Find P(F), if P(E) =0.35 and P(EUF)=0.6.

Ans: $P(EUF) = P(E) + P(F) - P(E \cap F)$

$= P(E) + P(F) - P(E).P(F)$

$0.6 = 0.35 + x - 0.35x; 0.25 = 0.65x; x = \frac{25}{65} = \frac{5}{13}$

7. A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them selected is 0.6. Find the probability that B is selected.

Ans: E: Selecting A F: Selecting B E and F are independent events.

$P(E) = 0.7, P[(E \cap F')U(E' \cap F)] = 0.6$

$P(E).P(F') + P(E').P(F) - P[(E \cap F') \cap (E' \cap F)] = 0.6$

$P(E)(1 - P(F)) + (1 - P(E))P(F) = 0.6$

$P(E) + P(F) - 2.P(E).P(F) = 0.6; 0.7 + x - 2(0.7).x = 0.6$

$0.1 = 0.4xx = \frac{1}{4} = P(F).$

8. A bag contains 3 white, 4 red and 5 black balls. Two balls are drawn at random. Find the probability that both balls are of different colours.

Ans:

P(both balls are of different colours) = 1 − P(both balls of same colour)

P(both balls of same colour) $= \frac{^3C_2}{^{12}C_2} + \frac{^4C_2}{^{12}C_2} + \frac{^5C_2}{^{12}C_2} = \frac{3}{66} + \frac{6}{66} + \frac{10}{66} = \frac{19}{66}$

P(both balls are of different colours) $= 1 - \frac{19}{66} = \frac{47}{66}$

9. An unbiased die is thrown thrice. Find the probability of getting at least 2 sixes.

Ans:

P(at least 2 sixes) = P(2 sixes) + P(3 Sixes)

$= 3.\frac{1}{6}.\frac{1}{6}.\frac{5}{6} + \frac{1}{6}.\frac{1}{6}.\frac{1}{6} = \frac{16}{216} = \frac{2}{27}$

10. A problem is given to A, B and C. The probabilities that they solve the problem

correctly are 1/3, 2/7 and 3/8 respectively. If they try to solve the problem simultaneously, find the probability that exactly one of them solve the problem.

Ans: $P(\text{exactly one solve}) = P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$$= \frac{1}{3}\cdot\frac{5}{7}\cdot\frac{5}{8} + \frac{2}{3}\cdot\frac{2}{7}\cdot\frac{5}{8} + \frac{2}{3}\cdot\frac{5}{7}\cdot\frac{3}{8} = \frac{75}{168}$$

SHORT ANSWER TYPE QUESTIONS

1. A and B throw a die alternately till one of them get a '6' and wins the game. Find their respective probabilities of winning, if A starts the game first.

Ans: S-Getting 6 F- not getting 6

$P(S)=p=1/6$ $P(F)=q=5/6$

$P(A - Wins) = p + qqp + qqqqp + \cdots\ldots$

$=1/6+(5/6)^2 1/6+(5/6)^4 1/6+\ldots.$

$$= \frac{1/6}{1-\frac{25}{36}} = \frac{6}{11}, \quad P(B\ \text{Wins}) = 1 - \frac{6}{11} = \frac{5}{11}$$

2. A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B is the event, "3 comes on the die". Find whether A and B are independent events or not.

Ans: S={(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}

A: H appears B: 3 on die

$P(A)=6/12=1/2$ $P(B)=2/12=1/6$ $P(A \text{ and } B)=1/12$ $P(A).P(B)=1/2.1/6=1/12=P(A \text{ and } B)$

Hence A and B are independent events.

3. There are two coins. One of them is a biased coin such that P(head): P(tail) is 1:3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin shows head, then find the probability that it is a biased coin

Ans:

A: Selecting Biased Coin B; Selecting fair coin C: Getting H

$P(A)=P(B)=1/2$, $P(C/A)=1/4$ (since ratio for head and tail is 1:3) $P(C/B)=1/2$

$P(C) = P(A).P(C/A)+P(B).P(C/B)=1/2.1/4+1/2.1/2=1/8+1/4=3/8$

$$P(B/C) = \frac{P(B)P(C/B)}{P(A)P(C/A)+P(B)P(C/B)} = \frac{1/4}{3/8} = 2/3$$

4. A letter is known to have come either from TATANAGAR or from CALCUTTA. What is the probability that on the envelope just two consecutive letters TA are visible?

Ans:

$P(A) = 1/2$, $P(B) = 1/2$,C-TA visible on the envelope.

$P(C/A)=2/8=1/4$, $P(C/B)=1/7$,

$$P(C) = P(A)P(C/A) + P(B)P(C/B) = \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{7} = 11/56$$

5. A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$ Find P(A) and P(B).

Ans:

$P(A)P(B') = 1/4$ & $P(A')P(B) = 1/6$ $P(A) = x$, $P(B) = y$

$x(1-y) = 1/4$ and $(1-x)y = 1/6$

on solving we get, $x - y = 1/12$ $x = y + 1/12$

On substituting, we get $12y^2 - 11y + 2 = 0$

$y = 1/4$ or $y = 2/3$, Corresponding $x = 1/3$ or $3/4$

$P(A) = 1/3$ $P(B) = 1/4$ (OR) $P(A) = 3/4$ $P(B) = 2/3$

6. There are 2000 scooter drivers, 4000 car drivers and 6000 truck drivers all insured. The probabilities of an accident involving a scooter, a car, a truck are 0.01, 0.03, and 0.15 respectively. What is the probability that one of the insured person meets with an accident.

Ans:

$P(A) = 1/6,\ P(B) = 1/3\ P(C) = 1/2$

D-the insured person meets with an accident.

$P(D/A)=1/100,\ P(D/B)=3/100,\ P(D/C)=15/100,$

$P(D) = P(A).P(D/A) + P(B).P(D/B) + P(C).P(D/C) = 13/150$

7. A bag contains 4 balls. Two balls are drawn at random. What is the probability that both drawn balls are white.

Ans: $A: 2W + 2nonwhite B: 3W + 1nonwhite C: 4W$

$P(A) = 1/3, P(B) = 1/3 P(C) = 1/3$ D-both drawn balls are White

$P(D/A) = {}^2C_2/{}^4C_2=1/6, P(D/B) = {}^3C_2/{}^4C_2=1/2, P(D/C) = {}^4C_2/{}^4C_2=1$

$P(D) = P(A).P(D/A) + P(B).P(D/B) + P(C).P(D/C) = \frac{1}{3}.\frac{1}{6}+\frac{1}{3}.\frac{1}{2}+\frac{1}{3}.1 = \frac{5}{9}$

8. Bag A contains 6 red and 5 blue balls and another bag B contains 5 red and 8 blue balls. A ball is drawn from bag A without seeing its colour and is put into the bag B. Then a ball is drawn from bag B at random. What is the probability that the ball drawn is blue in colour.

Ans:

A:Red$from A(6R + 5B) B: Blue from A(6R + 5B)$

C:getting Blue ball from bag B after transferring.

$P(A) = 6/11, P(B) = 5/11$

$Case: I A \cap C$

$$P(A \cap C) = P(A).P(C/A) = \frac{6}{11}.\frac{8}{14} = \frac{48}{154}$$

$Case: II B \cap C$

$$P(B \cap C) = P(B).P(C/B) = \frac{5}{11}.\frac{9}{14} = \frac{45}{154}$$

Reqd.Probability$=\frac{48}{154} + \frac{45}{154} = \frac{93}{154}$

CASE STUDY BASED QUESTIONS

1. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 3/10, 1/5, 1/10 and 2/5. The probabilities that he will be late are 1/4, 1/3, 1/12 and 1/10 if he comes by cab, metro, bike and other means of transport respectively.

(a) What is the probability that the doctor arrived late?

(b) When the doctor arrives late, what is the probability that he comes by metro?

Ans: $(a) A: Cab B: Metro C: Bike D: other$

$E: Late arrival$

$P(E) = P(A).P(E/A) + P(B).P(E/B) + P(C).P(E/C) + P(D).P(E/D)$

$= \frac{3}{10}.\frac{1}{4} + \frac{1}{5}.\frac{1}{3} + \frac{1}{10}.\frac{1}{12} + \frac{2}{5}.\frac{1}{10} = \frac{114}{\ldots}$

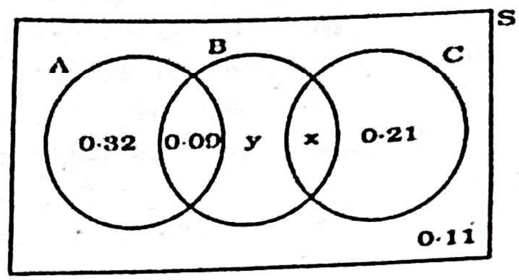
$$(b)P(B/E) = \frac{P(B).P(E/B)}{P(E)} = \frac{1/15}{114/600} = 40/114 = 20/57$$

2. The Venn diagram below represents the probabilities of three different types of Yoga A, B and C performed by the people of a society. Further, it is given that the probability of a member performing type C yoga is 0.44.

(i) Find the value of x. (ii) Find the value of y

(iii) (a) Find P(C/B) (OR)

(b) Find the probability that a randomly selected person of the society does Yoga type A or B but not C.



Ans: (i) x=0.44-0.21=0.23 (ii) y=1-0.96=0.04

$$\text{(iii) } P(C/B) = \frac{P(C\cap B)}{P(B)} = \frac{0.23}{0.36} = \frac{23}{36}(OR)$$

$$P(A\ OR\ B\ not\ C) = 0.32 + 0.09 + y = 0.41 + 0.04 = 0.45$$

3. Two cards are drawn from a well shuffled pack of 52 cards without replacement.

(a) What is the probability that one is a red queen and the other is a king of black colour

(b) one shows a prime number (from 2 to 10) and other is a face card(other than Ace)

Ans:

$(a) A = Red\ queen in$ 1st attempt B=Black King in 2nd Attempt

C=Black King in 1st Attempt

D= $Red\ queen in$ 2nd attempt

$$\text{Reqd Prob=P}(A\cap B)\text{+P}(C\cap D) = \frac{2}{52}\cdot\frac{2}{51} + \frac{2}{52}\cdot\frac{2}{51} = \frac{2}{663}$$

$(b) A = PrimeNumberin$ 1st attempt

B=face card in 2nd Attempt C=Face card in 1st Attempt D= $PrimeNumberin$ 2nd attem

$$\text{ReqdProb=P}(A\cap B)\text{+P}(C\cap D) = \frac{16}{52}\cdot\frac{12}{51} + \frac{12}{52}\cdot\frac{16}{51} = \frac{32}{221}$$

LONG ANSWER TYPE QUESTIONS

1. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/5 is the probability that he knows the answer and 2/5 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/3. What is the probability that the student knows the answer, given that he answered it correctly?

Ans: A: Knows the answer B: Guesses the answer E: Answered Correctly

P(A)=3/5, P(B)=2/5 P(E/A)=1, P(E/B)=1/3

By Bayes theorem,

$$P(A/E) = \frac{P(A).P(E/A)}{P(A).P(E/A) + P(B).P(E/B)} = \frac{\frac{3}{5}.1}{\frac{3}{5}.1 + \frac{2}{5}.\frac{1}{3}} = \frac{3}{11}$$

2. Three bags contains a number of red and white balls as follow:

Bag-1 contains 3 red balls, bag-2 contains 2 red and 1 white ball, bag-3 contains 3 white balls. The probability that bag -I will be chosen and a ball is selected from it is i/6, i=1,2,3.

(a) What is the probability that a red ball will be selected

(b) What is the probability that a white ball will be selected

Ans:

(a)A,B,C are events selecting bag-1,bag-2,bag-3respectively,D-selecting red colour ball

$P(D/A)=1, P(D/B)=2/3, P(D/C)=0$

$P(D)=P(A).P(D/A)+P(B).P(D/B)+P(C).P(D/C)$

$=\frac{1}{6}.1+\frac{2}{6}.\frac{2}{3}+0=\frac{7}{18}$ (b)$P(Whiteball)=1-\frac{7}{18}=\frac{11}{18}$

3. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

Ans:$A: twoheaded B: Biased(75\%H)coin C: biased(40\%T)$

$P(A)=P(B)=P(C)=1/3 H: getting H$

$P(H/A)=1, P(H/B)=75/100, P(H/C)=60/100$

$$P(A/H)=\frac{P(A).P(H/A)}{P(A).P(H/A)+P(B).P(H/B)+P(C).P(H/C)}=\frac{100}{235}$$

4 A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Ans: $A=6appears, B=nonsixappears$

$P(A)=1/6, P(B)=5/6 C: Reporting 6$

$P(C/A)=3/4, P(C/B)=1/4; P(C)=P(A).P(C/A)+P(B).P(C/B)$

$=\frac{1}{6}.\frac{3}{4}+\frac{5}{6}.\frac{1}{4}=\frac{8}{24}$

$$P(A/C)=\frac{P(A).P(C/A)}{P(A).P(C/A)+P(B).P(C/B)}=\frac{\frac{1}{6}.\frac{3}{4}}{\frac{8}{24}}=\frac{3}{8}$$

5 Assume that the chances of a patient having a heart attack are 40%. It is also assumed that meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options and patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

Ans;

$A-Meditation\&Yoga B-takesdrug C-SuffersheartAttack$

$P(A)=P(B)=\frac{1}{2} P(C/A)=\frac{70}{100}.\frac{40}{100}, P(C/B)=\frac{75}{100}.\frac{40}{100}$

$$P(A/C)=\frac{\frac{1}{2}.\frac{70}{100}.\frac{40}{100}}{\frac{1}{2}.\frac{70}{100}.\frac{40}{100}+\frac{1}{2}.\frac{75}{100}.\frac{40}{100}}=\frac{14}{29}$$

6. Bhavani is going to pay a game of chess against one of four opponents in an inter-college sports competition. Each opponent is equally likely to be paired against her. The table below shows the chances of Bhavani losing, where paired against each opponent.

OPPONENT	BHAVANI'S CHANCE OF LOSSING
OPP-1	12%
OPP-2	60%
OPP-3	X%
OPP-4	84%

If the probability that Bhavani losses the game that day is ½. find the probability for Bhavani to be losing when paired against opponent 3.

Sol A,B,C,D be events, respectively Bhavani game with Opp-1,2,3,4 . E be the vent Bhavani Loses the game.

Given P(E)=1/2

P(A)=P(B)=P(C)=P(D)=1/4.

P(E/A)=12/100, P(E/B)=60/100, P(E/C)=x/100,P(E/D)=84/100

P(A)=1/2

$$1/2 = P(A).P(E/A) + P(B).P(E/B) + +P(C).P(E/C) + P(D).P(E/D)$$

$$1/2 = \frac{12 + 60 + X + 84}{400}; X = 44p(E/C) = 44\%$$

7. Three persons A , B, C apply for a job a manager in a company. Chances of their selection are in the ratio 1:2:3. The probability that A,B and C can introduce chances to increase the profits of a company are 0.8,0.5 and 0.3 respectively. If increase in the profit does not take place, find the probability that it is due to the appointment of A.

P(A)=1/7, P(B)=2/7, P(C)=4/7

E-not increase the profits

P(E/A)=0.2, P(E/B)=0.5, P(E/C)=0.7

$$P(E/A) = \frac{P(A).P(E/A)}{= P(A).P(E/A) + P(B).P(E/B) + +P(C).P(E/C)} = \frac{2/70}{40/70} = 1/20$$

8. There are two bags I and II, Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to bag II and then a ball is drawn randomly from bag II. If the ball so drawn is found to be black in colour then find the probability that the transferred ball is also black.

A-transferred ball is Red B-transferred ball is black

C: ball from bag II after transferred , is black.

P(A)=3/8, P(B)= 5/8

P(C/A)=3/8 PC/B)=4/8

$$P(C/B) = \frac{P(A).P(C/A)}{P(A).P(C/A) + P(B).P(C/B)} = \frac{\frac{3}{8}\cdot\frac{3}{8}}{\frac{3}{8}\cdot\frac{3}{8} + \frac{5}{8}\cdot\frac{4}{8}} = \frac{9}{29}$$